



Hanoi Pedagogical University 2



Institute of Mathematics, VAST

INTERNATIONAL CONFERENCE

**OPTIMIZATION AND CONTROL THEORY
WITH APPLICATIONS**



PROGRAM & ABSTRACTS

Hanoi Pedagogical University 2
24/08/2023 - 26/08/2023



INTERNATIONAL CONFERENCE
OPTIMIZATION AND CONTROL THEORY WITH APPLICATIONS

PROGRAM AND ABSTRACTS



Hanoi Pedagogical University 2
24/08/2023 - 26/08/2023

International Conference
Optimization and Control Theory with Applications
24/08/2023 - 26/08/2023

Sponsors

- Hanoi Pedagogical University 2
- Institute of Mathematics, Vietnam Academy of Science and Technology

Location

Hanoi Pedagogical University 2, Nguyen Van Linh Street, Xuan Hoa Ward, Phuc Yen City, Vinh Phuc Province, Vietnam.

Organizing Committee

- **Nguyen Quang Huy** (Hanoi Pedagogical University 2, Vietnam)
- **Doan Thai Son** (Institute of Mathematics, VAST, Vietnam)
- **Bui Trong Kien** (Institute of Mathematics, VAST, Vietnam)
- **Cung The Anh** (Hanoi National University of Education, Vietnam)
- **Lam Quoc Anh** (Can Tho University, Vietnam)
- **Pham Ngoc Anh** (Posts and Telecommunications Institute of Technology, Vietnam)
- **Tran Van Bang** (Hanoi Pedagogical University 2, Vietnam)
- **Le Van Hien** (Hanoi National University of Education, Vietnam)
- **Nguyen Hoai Nam** (Hanoi University of Science and Technology, Vietnam)
- **Nguyen Huyen Muoi** (Institute of Mathematics, VAST, Vietnam)
- **Le Quang Thuy** (Hanoi University of Science and Technology, Vietnam)
- **Hoang Ngoc Tuan** (Hanoi Pedagogical University 2, Vietnam)
- **Nguyen Van Tuyen** (Hanoi Pedagogical University 2, Vietnam)
- **Le Hai Yen** (Institute of Mathematics, VAST, Vietnam)

Scientific Committee

- **Prof. Dinh Nho Hao** (Institute of Mathematics, VAST, Vietnam)
- **Assoc. Prof. Nguyen Quang Huy** (Hanoi Pedagogical University 2, Vietnam)
- **Prof. Phan Quoc Khanh** (Ton Duc Thang University, Vietnam)
- **Dr. Bui Trong Kien** (Institute of Mathematics, VAST, Vietnam)
- **Prof. Dinh The Luc** (University of Avignon, France)
- **Prof. Le Dung Muu** (Thang Long University, Vietnam)
- **Prof. Nguyen Mau Nam** (Portland State University, USA)
- **Assoc. Prof. Huynh Van Ngai** (Quy Nhon University, Vietnam)
- **Prof. Vu Ngoc Phat** (Institute of Mathematics, VAST, Vietnam)
- **Prof. Hoang Xuan Phu** (Institute of Mathematics, VAST, Vietnam)
- **Prof. Nguyen Doan Phuoc** (Hanoi National University of Science and Technology, Vietnam)
- **Prof. Nguyen Khoa Son** (Institute of Mathematics, VAST, Vietnam)
- **Prof. Nguyen Dong Yen** (Institute of Mathematics, VAST, Vietnam)

Local Organizing Committee

- **Bui Kien Cuong** (Hanoi Pedagogical University 2, Vietnam)
- **Hoang Ngoc Tuan** (Hanoi Pedagogical University 2, Vietnam)
- **Tran Van Bang** (Hanoi Pedagogical University 2, Vietnam)
- **Nguyen Van Dung** (Hanoi Pedagogical University 2, Vietnam)
- **Nguyen Huu Hoa** (Hanoi Pedagogical University 2, Vietnam)
- **Nguyen Huy Hung** (Hanoi Pedagogical University 2, Vietnam)
- **Nguyen Hoang Ngoc** (Hanoi Pedagogical University 2, Vietnam)
- **Tran Minh Tuoc** (Hanoi Pedagogical University 2, Vietnam)
- **Nguyen Van Tuyen** (Hanoi Pedagogical University 2, Vietnam)

Schedule of the International Conference Optimization and Control Theory with Applications

Morning, Thursday, August 24, 2023
A1 Hall - Hanoi Pedagogical University 2

08:00 – 08:30 **Registration**

08:30 – 08:40 **Opening Ceremony**

Chair: Nguyen Dong Yen

08:40 – 09:40 **Boris S. Modukhovich** (Wayne State University, USA)
Optimal control of perturbed sweeping processes with applications

09:40 – 10:00 **Coffee Break**

Chair: Ngai-Ching Wong

10:00 – 10:40 **Samir Adly** (University of Limoges, France)
*Riemannian gradient structure in accelerated dynamics
with dry friction: Time scaling and averaging of doubly
nonlinear evolution equations*

10:40 – 11:20 **Do Sang Kim** (Pukyong National University, South Korea)
*Approximate solutions of nonsmooth semi-infinite
multiobjective optimization problems*

11:20 – 12:00 **Nguyen Dong Yen** (Institute of Mathematics, VAST, Vietnam)
On the multi-source Weber problem

12:00 – 12:10 **Souvenir Photograph**

12:10 – 14:00 **Lunch Break**

Afternoon, Thursday, August 24, 2023
A1 Hall - Hanoi Pedagogical University 2

SECTION 1

Chair: Nguyen Van Tuyen

- 14:00 – 14:20 **Pham Tien Son** (Dalat University, Vietnam)
Lipschitz continuous selections of solutions and implicit function theorems for locally Lipschitz equations
- 14:20 – 14:40 **Nguyen Nang Tam** (Duy Tan University, Vietnam)
A non-polyhedral extension of the Frank-Wolfe theorem to cubic optimization
- 14:40 – 15:00 **Le Hai Yen** (Institute of Mathematics, VAST, Vietnam)
An extragradient algorithm for quasiconvex equilibrium problems without monotonicity
- 15:00 – 15:20 **Coffee Break**

Chair: Pham Tien Son

- 15:20 – 15:40 **Nguyen Van Tuyen** (Hanoi Pedagogical University 2, Vietnam)
Optimality conditions and sensitivity analysis in parametric nonconvex minimax programming
- 15:40 – 16:00 **Ramakant Bhardwaj** (Amity University, India)
Fixed point results on soft metric space
- 16:00 – 16:20 **Izhar Uddin** (Aligarh Muslim University, India)
Modified proximal point algorithm and its applications
- 16:20 – 16:40 **Pham Duy Khanh** (Ho Chi Minh City University of Education, Vietnam)
Lipschitz continuity of convex functions
- 16:40 – 17:00 **Vo Si Trong Long** (University of Science, Vietnam National University, Ho Chi Minh City, Vietnam)
Generalized relative interiors and generalized convexity in infinite dimensions

Evening, Thursday, August 24, 2023

19:00 **Banquet**

Afternoon, Thursday, August 24, 2023
Meeting Room 2, A1 House - Hanoi Pedagogical University 2

SECTION 2

Chair: Phan Thanh Nam

- 14:00 – 14:20 **Vu Huu Nhu** (Phenikaa University, Vietnam)
Levenberg-Marquardt method for non-smooth ill-posed inverse problems in Banach spaces
- 14:20 – 14:40 **Nguyen Hai Son** (Hanoi University of Science and Technology, Vietnam)
Solution stability of parametric boundary control problems
- 14:40 – 15:00 **Purvee Bhardwaj** (APS University, Rewa (MP), India)
Stability of intermetallic compound through geometry optimization
- 15:00 – 15:20 **Coffee Break**

Chair: Lam Quoc Anh

- 15:20 – 15:40 **Nguyen Huyen Muoi** (Institute of Mathematics, VAST, Vietnam)
 H_∞ finite time stability for singular systems with delay
- 15:40 – 16:00 **Tran Van Tuan** (Hanoi Pedagogical University 2, Vietnam)
On temporal regularity of solutions for a nonlinear time-delayed fractional reaction-diffusion equation
- 16:00 – 16:20 **Nguyen Quoc Tuan** (Hanoi Pedagogical University 2, Vietnam)
Locally Lipschitz continuity of the solution map for a parametric semilinear elliptic optimal control problem
- 16:20 – 16:40 **Nguyen Phuong Dong** (Hanoi Pedagogical University 2, Vietnam)
A new Takagi-Sugeno fuzzy system approach for fuzzy state feedback controller design and its application to malware propagation on heterogeneous complex network
- 16:40 – 17:00 **Cao Thanh Tinh** (University of Information Technology, Ho Chi Minh City, Vietnam)
Stability and robust stability of non-autonomous linear differential equations with infinite delay

Evening, Thursday, August 24, 2023

19:00 **Banquet**

Morning, Friday, August 25, 2023
A1 Hall - Hanoi Pedagogical University 2

Chair: Boris S. Modukhovich

- 08:00 – 09:00 **Arnd Rösch** (Duisburg-Essen University, Germany)
A semilinear parabolic problem with a directional sparsity functional
- 09:00 – 09:40 **Qamrul Hasan Ansari** (Aligarh Muslim University, India)
A projection-type hybrid conjugate gradient method for vector optimization problems with application to an optimal control problem
- 09:40 – 10:00 **Coffee Break**

Chair: Samir Adly

- 10:00 – 10:40 **Ngai-Ching Wong** (National Sun Yat Sen University, Kaohsiung Kaohsiung, Taiwan)
The no trade principle and the characterization of compact beliefs
- 10:40 – 11:20 **Daya Ram Sahu** (Banaras Hindu University, India)
Accelerated extragradient methods and further acceleration for variational inequality problems via the property (A)
- 11:20 – 12:00 **Phan Thanh Nam** (Quy Nhon University, Vietnam)
State bounding for positive time-delay systems perturbed by bounded disturbances
- 12:00 – 12:10 **Closing session**

Afternoon, Friday, August 25, 2023
Excursion (*Truc Lam Tay Thien Zen Monastery*)

Saturday, August 26, 2023
Excursion (*Giong Temple*)

ABSTRACTS



Riemannian gradient structure in accelerated dynamics with dry friction: Time scaling and averaging of doubly nonlinear evolution equations

Samir Adly¹

Abstract: In a Hilbert framework, with the aim of convex differentiable optimization, we analyze the long-time behavior of inertial dynamics with dry friction. We rely on the general acceleration method recently developed by Attouch, Bot and Nguyen, which consists of applying the time scaling and then averaging method to a first order in time continuous differential equation. In our approach, we start from a doubly nonlinear first-order evolution equation involving two potentials: one is the differentiable function f to be minimized, which acts on the state of the system via its gradient, and the other is the dry friction potential $\varphi(x) = r\|x\|$ which acts on the velocity vector via its subdifferential.

¹ XLIM UMR-CNRS 7252, Université de Limoges
870 60 Limoges, France
Email: samir.adly@unilim.fr

A projection-type hybrid conjugate gradient method for vector optimization problems with application to an optimal control problem

Qamrul Hasan Ansari¹

Abstract: In this talk, we propose a projection-type hybrid conjugate gradient method for solving vector optimization problems. It is an extension of the Hestenes–Stiefel and Dai-Yuan projection type hybrid conjugate gradient method for vector-valued case. We show that the proposed method generates the directions which satisfy the sufficient descent condition under the strong Wolfe line search for vector-valued functions. The global convergence of the proposed scheme is studied without any convexity assumption. To demonstrate the strength of the method and its practical applicability, we apply it to a set of commonly used test problems. In addition, we compare the performance of the proposed method with two standard conjugate gradient methods through relative efficiency. As an application, the proposed method is applied to an SIR epidemiological optimal control model with vaccination and treatment as their controls. Towards this aim, we propose a multi-objective optimization approach to find the optimal strategies for minimizing both the number of infected individuals and the implementation cost of the control strategies.

¹ Department of Mathematics, Aligarh Muslim University
Aligarh 202 002, India
Email: qhansari@gmail.com

Stability of intermetallic compound through geometry optimization

Purvee Bhardwaj^{1,2}

Abstract: This study examines the structural electronic characteristics of B2 rare earth intermetallic complexes. According to the Perdew-Burke-Emzerhof (GGA-PBE) approach, the generalised gradient approximation (PBE) method has been used to perform the electronic computations. Current intermetallics crystallise in a CsCl-type structure, according to enthalpy calculations. This technique is applicable for managing all styles of first-principles-based local optimizations. Electronic band structure and state density are explored in relation to the stability of the current molecule. The predicted equilibrium structural parameters, such as the bulk modulus, lattice constants, and its derivative, are in strong agreement with the outcomes.

¹ Department of Physics, APS University,
Rewa, (MP) 468003, India

² Faculty of Science, Rabindranath Tagore (RNTU) University,
Bhopal (MP) 464993
Email: purvee.bhardwaj@aisectuniversity.ac.in

Fixed point results on soft metric spaces

Ramakant Bhardwaj¹, Rakhi Namdev², Rashmi Tiwari³, Deb Sarkar⁴

Abstract: In this paper, some new inequalities have been introduced using control functions. Using these inequalities some new results of fixed point are established for soft metric spaces. The results are supported by suitable examples and also some consequences are given.

^{1, 4} Department of Mathematics, Amity University,
West Bengal-700135, India

² Department of Mathematics, Govt. Narmada College,
Narmada Puram, M.P. India

³ Department of Mathematics, Govt. MGM College,
Itarsi, M.P. India
Email: ¹rkbhardwaj100@gmail.com, ²rakhinamdev16@gmail.com,
³rashmi.v.tiwari@gmail.com, ⁴debsarkar1996@gmail.com

A new Takagi-Sugeno fuzzy system approach for fuzzy state feedback controller design and its application to malware propagation on heterogeneous complex network

Nguyen Phuong Dong^{1, 2}

Abstract:

Nowadays, digital transformation has brought many great changes and is becoming an essential part of real life, however, it also goes along with a considerable likelihood of being targeted in cyberattack. For better analyzing and predicting the attack of malware program on real-world networks, we propose to study a controlled fractional network-based SCIRS malware propagation model and its stabilization problem based on fractional interconnected Takagi-Sugeno fuzzy system. A fuzzy state feedback controller is proposed to asymptotically stabilize the unstable malware-free equilibrium of the proposed malware propagation model and then, we establish sufficient conditions in terms of linear matrix inequalities. The effectiveness of proposed approach is illustrated by a case study of SCIRS malware propagation model on heterogeneous complex network.

¹ Department of Mathematics, Hanoi Pedagogical University 2,
Nguyen Van Linh Street, Xuan Hoa Ward, Phuc Yen City,
Vinh Phuc Province, Vietnam

² Graduate University of Science and Technology, Vietnam Academy
of Science and Technology
18 Hoang Quoc Viet, 10307 Cau Giay, Hanoi, Vietnam
Email: nguyenvhuongdong@hpu2.edu.vn

Approximate solutions of nonsmooth semi-infinite multiobjective optimization problems

Do Sang Kim¹

Abstract: In this talk, we establish fuzzy necessary optimality conditions for (weak) ε -Pareto solutions, and exact necessary optimality conditions for (weak) quasi ε -Pareto solutions to a nonsmooth semi-infinite multiobjective optimization problems by means of employing some advanced tools of variational analysis and generalized differentiation. In addition, we state a dual problem formulated in an approximate form to the reference problem and explore duality relations between them. We also design some examples to analyze and illustrate the obtained results.

¹ Department of Applied Mathematics, Pukyong National University,
Busan, 48513, Korea
Email: dskim@pknu.ac.kr

Lipschitz continuity of convex functions

Pham Duy Khanh¹

Abstract: We provide some necessary and sufficient conditions for a proper lower semicontinuous convex function, defined on a real Banach space, to be locally or globally Lipschitz continuous. Our criteria rely on the existence of a bounded selection of the subdifferential mapping and the intersections of the subdifferential mapping and the normal cone operator to the domain of the given function. Moreover, we also point out that the Lipschitz continuity of the given function on an open and bounded (not necessarily convex) set can be characterized via the existence of a bounded selection of the subdifferential mapping on the boundary of the given set and as a consequence it is equivalent to the local Lipschitz continuity at every point on the boundary of that set. Our results are applied to extend a Lipschitz and convex function to the whole space and to study the Lipschitz continuity of its Moreau envelope functions.

¹ Department of Mathematics, Ho Chi Minh City University of Education
Ho Chi Minh City, Vietnam
Email: pdkhanh182@gmail.com

Generalized relative interiors and generalized convexity in infinite dimensions

Vo Si Trong Long^{1,2}, Boris S. Modukhovich³ and Nguyen Mau Nam⁴

Abstract: This paper focuses on investigating generalized relative interior notions for sets in locally convex topological vector spaces with particular attentions to graphs of set-valued mappings and epigraphs of extended-real-valued functions. We introduce, study, and utilize a novel notion of *quasi-near convexity* of sets that is an infinite-dimensional extension of the widely acknowledged notion of near convexity. Quasi-near convexity is associated with the quasi-relative interior of sets, which is investigated in the paper together with other generalized relative interior notions for sets, not necessarily convex. In this way, we obtain new results on generalized relative interiors for graphs of set-valued mappings in convexity and generalized convexity settings.

¹ Faculty of Mathematics and Computer Science, University of Science, Ho Chi Minh City, Vietnam.

² Vietnam National University, Ho Chi Minh City, Vietnam
Email: vstlong@hcmus.edu.vn

³ Department of Mathematics, Wayne State University,
Detroit, Michigan 48202, USA
Email: boris@math.wayne.edu

⁴ Fariborz Maseeh Department of Mathematics and Statistics,
Portland State University, Portland, OR 97207, USA
Email: mnn3@pdx.edu

H_∞ finite time stability for singular systems with delay

Nguyen Huyen Muoi¹

Abstract: In this talk, we present H_∞ finite time control for singular linear differential system with delay. Based on the Lyapunov-like functional method, new delay-dependent sufficient conditions such that the system is regular, impulse-free and H_∞ finite-time stable are developed in terms of solutions of some linear matrix inequalities.

References

- [1] F. Amato, R. Ambrosino, M. Ariola, C. Cosentino, G. De Tommasi, *Finite-time stability and control*, Lecture Notes in Control and In Information Sciences, **453**, Springer London, 2014.
- [2] S.L Campbell, *Singular Systems of Differential Equations*, Pitman, London, 1980.
- [3] P. Dorato, Short time stability in linear time-varying systems, *In Proc IRE Int Convention Record*, Part 4(1961), 83–87.
- [4] P. Gahinet, A. Nemirovskii, A.J. Laub, M. Chilali, *LMI Control Toolbox For use with MATLAB*, The MathWorks, Inc, 1995.
- [5] G. Kamenkov, On stability of motion over a finite interval of time, *Journal of Applied Math. and Mechanics*, **17**(1953), 529–540.

¹ Department of Optimization and Control,
Institute of Mathematics, Vietnam Academy of Science and Technology
18 Hoang Quoc Viet, 10307 Cau Giay, Hanoi, Vietnam
Email: nhmuoi@math.ac.vn

Optimal control of perturbed sweeping processes with applications

Boris S. Modukhovich¹

Abstract: This talk is devoted to a new class of dynamic optimization problems for unbounded discontinuous differential inclusions governed by controlled versions of the Moreau's sweeping process. To study such problems, we develop a refined method of discrete approximations with establishing its wellposedness and strong convergence. This approach, married to advanced tools of first-order and secondorder variational analysis and generalized differentiation, allows us to derive adequate collections of necessary optimality conditions for local minimizers, first in discrete-time problems and then in the original continuous-time controlled sweeping process by passing to the limit. The obtained necessary optimality conditions are applied to practical models of robotics and traffic equilibria.

The talk is mainly based on joint research with Giovanni Colombo and Dao Nguyen.

¹ Department of Mathematics,
Wayne State University,
Detroit, MI, USA
Email: boris@math.wayne.edu

State bounding for positive time-delay systems perturbed by bounded disturbances

Phan Thanh Nam¹

Abstract: Disturbance is frequently unavoidable in real-life systems. In general, the disturbance is unknown but it is often assumed to be bounded by a known bound. Under the effect of the disturbance, it is hard to obtain the exact value of the system's state vector. In this talk, we give a discussion, including an overview and recent developments, on the state bounding problem for classes of positive time-delay systems with bounded disturbances.

¹ Department of Mathematics , Quinhon University
170 An Duong Vuong Road, Binh Dinh, Vietnam
Email: phanthanhnam@qnu.edu.vn

Levenberg-Marquardt method for non-smooth ill-posed inverse problems in Banach spaces

Vu Huu Nhu¹

In this talk, we present a Levenberg–Marquardt method for ill-posed inverse problems in Banach spaces [1]. There, the regularization terms are uniformly convex on bounded sets and the forward mapping might not be Gâteaux differentiable as well as the image space is unnecessarily reflexive. The method therefore extends the one proposed by Jin and Yang in [2] for the smooth inverse problem setting with globally uniformly convex regularization terms. We prove a novel convergence analysis of the proposed method under some standing assumptions, in particular, the generalized tangential cone condition and a compactness assumption. All these assumptions are fulfilled when investigating the identification of the heat source for semilinear elliptic boundary-value problems with a Robin boundary condition, a heat source acting on the boundary, and a possibly non-smooth nonlinearity. Therein, the Clarke sub-differential of the non-smooth nonlinearity is employed to construct the family of bounded operators that is a replacement for the non-existing Gâteaux derivative of the forward mapping. The efficiency of the proposed method is illustrated with a numerical example.

References

- [1] V. H. Nhu, Levenberg–Marquardt method for ill-posed inverse problems with possibly non-smooth forward mappings between Banach spaces. *Inverse Problems* **38** (2022) 015007.
- [2] Q. Jin and H. Yang, Levenberg–Marquardt method in Banach spaces with general convex regularization terms. *Numer. Math.* **133** (2016) 655–84.

¹ Faculty of Fundamental Sciences,
Phenikaa University,
Yen Nghia, Ha Dong, Hanoi 12116, Vietnam
Email: nhu.vuhuu@phenikaa-uni.edu.vn

A semilinear parabolic problem with a directional sparsity functional

Arnd Rösch¹

Abstract: We consider a semilinear parabolic optimal control problem with a functional promoting directional sparsity. The optimal controls will be sparse in space but not in time. We show different theoretical results as existence of optimal solutions, optimality conditions, uniqueness of Lagrange multipliers, and sparsity properties of optimal controls. Moreover, we discuss the discretization of such problems by a Finite Element Method and show approximation rates.

¹ Fakultät für Mathematik, Universität Duisburg-Essen
Thea-Leymann-Straße 9 D-45127 Essen
Email: arnd.roesch@uni-due.de

Accelerated extragradient methods and further acceleration for variational inequality problems via the property (A)

Daya Ram Sahu¹

Abstract: One of the goals of this talk is to explore some important features of the property (A) of an operator/a family of nonlinear operators for solving nonlinear problems, inclusion problems and optimization problems. The main strategy of this talk is intended to speed up the convergence of the inertial Mann iterative method and further, speed up it through the normal S-iterative methodology for a certain class of nonexpansive type operators that are linked with variational inequality problems. There is an interesting fact that the relaxed inertial normal S-iterative extragradient methods do influence much more on convergence behaviour. Our new operator theoretic approach permits us to settle down the difficulty of unification of Korpelevich's extragradient method, Tseng's extragradient method, and subgradient extragradient method for solving variational inequality problems through an algorithmic operator, which enjoys the following:

- the seed operator has the property (A) with respect to the algorithmic operator.

¹ Department of Mathematics, Banaras Hindu University
Varanasi 221005, India
Email: drsahudr@gmail.com

Lipschitz continuous selections of solutions and implicit function theorems for locally Lipschitz equations

Pham Tien Son¹

Abstract: In this work, we first give affirmative answers to two open questions of nonlinear analysis, concerning the existence of a Lipschitz single-valued local selection of the right inverse to a locally Lipschitz map. Then we prove an implicit function theorem for equations, which defined by locally Lipschitz continuous maps with parameters.

¹ Department of Mathematics, Dalat University
1 Phu Dong Thien Vuong, Dalat, Vietnam
Email: sonpt@dhu.edu.vn

Solution stability of parametric boundary control problems

Nguyen Hai Son¹

Abstract: In this talk, we present the solution stability of a parametric boundary control problem governed by semilinear elliptic equations with finite mixed pointwise constraints. We show that under the strictly nonnegative second-order optimality conditions, the solution map is locally Hölder continuous in L^∞ -norm of control variables.

¹ School of Applied Mathematics and Informatics,
Hanoi University of Science and Technology,
1 Dai Co Viet, Hanoi, Vietnam
Email: son.nguyenhai1@hust.edu.vn

A non-polyhedral extension of the Frank-Wolfe theorem to cubic optimization

Tran Van Nghi¹, Nguyen Nang Tam²

Abstract: In 1956, Frank and Wolfe proved that a quadratic function which is bounded from below on a nonempty polyhedral convex set attains its infimum there. In 1982, Andronov, Belousov and Shironin extended Frank-Wolfe result to the case of cubic polynomials. In this note, we propose a non-polyhedral extension of Frank-Wolfe Theorem to nonconvex cubic programming problems.

References

- [1] V. G. Andronov, E.G. Belousov, V.M. Shironin, On solvability of the problem of polynomial programming (In Russian). *Izvestija Akadem Nauk SSSR, Tekhnicheskaja Kibernetika* **4** (1982) 194–197.
- [2] M. Frank, P. Wolfe, An algorithm for quadratic Programming. *Naval Research Logistics Quaterly* **3** (1956) 95–110.
- [4] S. T. Dinh, H. V. Ha, T. S. Pham, A Frank-Wolfe type theorem for nondegenerate polynomial programs. *Math. Program.* **147** (2014) 519–538.
- [5] V. T. Hieu, On the solution existence and stability of polynomial optimization problems. *Optim. Lett.* **16** (2022) 1513–1529.
- [6] D. Klatte, On a Frank-Wolfe type theorem in cubic optimization. *Optimization* **68** (2019) 539–547.
- [7] T. V. Nghi, N. N. Tam, A Frank–Wolfe-type theorem for cubic programs and solvability for quadratic variational inequalities. *J. Optim. Theory Appl.* **187** (2020) 448–468

¹ Department of Mathematics, Hanoi Pedagogical University 2.
 Nguyen Van Linh Street, Xuan Hoa Ward, Phuc Yen City,
 Vinh Phuc Province, Vietnam
Email: tranvannghi@hpu2.edu.vn; nghitv87@gmail.com

² Institute of Theoretical and Applied Research, Duy Tan University, Hanoi, Vietnam;
 Faculty of Natural Sciences, Duy Tan University, Da Nang, Vietnam
Email: nguyennangtam@duytan.edu.vn; nngtam@gmail.com

Locally Lipschitz continuity of the solution map for a parametric semilinear elliptic optimal control problem

Nguyen Quoc Tuan¹

Abstract: This talk is concerned with the stability of minimizers to a parametric optimal control problem governed by semilinear elliptic equations with mixed pointwise control-state constraints.

Let Ω be an open and bounded domain in \mathbf{R}^n with the boundary $\partial\Omega$ be of class $C^{1,1}$ and $n = 2, 3$. We consider the following parametric optimal control problem governed by semilinear elliptic equations with mixed pointwise control-state constraints: find, for each parameter $\lambda \in L^\infty(\Omega)$, a control function $u \in L^2(\Omega)$ and a corresponding state function $y \in H^2(\Omega) \cap H_0^1(\Omega)$ which minimize the cost functional

$$J(y, u, \lambda) = \int_{\Omega} \left(\eta(x, y(x), \lambda(x)) + \zeta(\lambda(x))u(x) + \mu(\lambda(x))u^2(x) \right) dx$$

s.t

$$Ay + h(\cdot, y, \lambda) = u + \lambda \text{ in } \Omega, \quad y = 0 \text{ on } \partial\Omega,$$

$$f(x, y(x), \lambda(x)) + \delta u(x) \geq 0 \text{ a.e. } x \in \Omega, \text{ and } u(x) \geq 0 \text{ a.e. } x \in \Omega,$$

where the operator A is defined by

$$Ay = - \sum_{i,j=1}^n D_j(a_{ij}(x)D_i y(x)) + a_0(x)y(x),$$

coefficients $a_{ij} \in C^{0,1}(\bar{\Omega})$ satisfy $a_{ij}(x) = a_{ji}(x)$, $a_0 \in L^\infty(\Omega)$, $a_0(x) \geq 0$ a.e. on Ω , $a_0(x) > 0$ on a set of positive measure, and there exists a positive number m_0 such that

$$m_0 \|\xi\|^2 \leq \sum_{i,j=1}^n a_{ij} \xi_i \xi_j \quad \forall \xi = (\xi_1, \xi_2, \dots, \xi_n) \in \mathbf{R}^n,$$

and $\delta > 0$; the functions η, h , and $f : \Omega \times \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ are Carathéodory functions and are of class C^2 with respect to second and third variables; $\zeta, \mu : \mathbf{R} \rightarrow \mathbf{R}$ are of class C^2 .

Under the strictly nonnegative second-order optimality condition assumption, we show that the solution map is locally Lipschitz continuous in L^2 -norm as well as in L^∞ -norm of the control variable.

¹ Department of Mathematics, Hanoi Pedagogical University 2
 Nguyen Van Linh Street, Xuan Hoa Ward, Phuc Yen City,
 Vinh Phuc Province, Vietnam
 Email: nguyenquoctuan@hpu2.edu.vn

On temporal regularity of solutions for a nonlinear time-delayed fractional reaction-diffusion equation

Trần Thị Thu¹, Trần Văn Tuấn²

Abstract: In this talk, we analyze the global existence and regularity in time of solutions to a fractional reaction-diffusion equation (FrRDE) involving delays and nonlinear perturbations in a bounded domain of \mathbf{R}^d . By establishing some regularity estimates of resolvent operator, we show results on the Hölder and C^1 -regularity in time of solutions for both time-delayed linear and nonlinear FrRDEs. Additionally, as an application of these results, several results on the existence, uniqueness and regularity of solutions to an identification problem subjected to the delay FrRDE and the additional measurements given at final time are also obtained.

¹ Department of Mathematics, Hanoi Pedagogical University 2
Nguyen Van Linh Street, Xuan Hoa Ward, Phuc Yen City,
Vinh Phuc Province, Vietnam
Email: tranthithu@hpu2.edu.vn

² Department of Mathematics, Hanoi Pedagogical University 2
Nguyen Van Linh Street, Xuan Hoa Ward, Phuc Yen City,
Vinh Phuc Province, Vietnam
Email: tranvantuan@hpu2.edu.vn

Stability and robust stability of non-autonomous linear differential equations with infinite delay

Cao Thanh Tinh¹

Abstract: In this talk, we first present novel explicit criteria for the differential equations with infinite delay. Some characterizations for the timeindependent exponential stability are also investigated. We then present a sufficient condition for the exponential stability of the equations subject to time-varying structured perturbations. These results lead to applications to the linear non-autonomous differential system with unbounded delay and the linear non-autonomous integro-differential equations with delay. Some examples are given to illustrate the obtained results.

¹ Vietnam National University-HCMC (VNU-HCM);
Department of Mathematics, University of Information Technology;
Thu Duc city, Vietnam.
Email: tinhct@uit.edu.vn

Optimality conditions and sensitivity analysis in parametric nonconvex minimax programming

Duong Thi Viet An, Nguyen Huy Hung, Dang Thi Ngoan and
Nguyen Van Tuyen¹

Abstract: In this talk, we perform optimality conditions and sensitivity analysis for parametric nonconvex minimax programming problems. Our aim is to study the necessary optimality conditions by using the Mordukhovich (limiting) subdifferential and to give upper estimations for the Mordukhovich subdifferential of the optimal value function in the problem under consideration. The optimality conditions and sensitivity analysis are obtained by using upper estimates for Mordukhovich subdifferentials of the maximum function. The results on optimality conditions are then applied to parametric multiobjective optimization problems. An example is given to illustrate our results.

¹ Department of Mathematics, Hanoi Pedagogical University 2
Nguyen Van Linh Street, Xuan Hoa Ward, Phuc Yen City,
Vinh Phuc Province, Vietnam
Email: nguyenvantuyen83@hpu2.edu.vn

Modified proximal point algorithm and its applications

Izhar Uddin¹

Abstract: Fixed point theory is a very applicable branch of mathematics. It is very good mixture of abstract and applied mathematics. Banach contraction theorem is one the most famous results in metric fixed point theory. Theory of nonexpansive mapping is very important in optimization problems. After discussing basic problems, we will focus on recent developments around the iterative fixed point theorem and its applications to optimization problem. The talk will be based on following recently published papers [1-2]:

References

[1] S. Khatoon, W. Cholamjiak, I. Uddin, A modified proximal point algorithm involving nearly asymptotically quasi-nonexpansive mappings. *Journal of Inequalities and Applications*. **2021** (1), (2021) 1–20.

[2] S. Khatoon, I. Uddin, M. Basarir, A modified proximal point algorithm for a nearly asymptotically quasi-nonexpansive mapping with an application. *Computational and Applied Mathematics*. **40** (7), (2021)

¹ Department of Applied Mathematics, Jamia Millia Islamia
New Delhi, India
Email: izharuddin1@jmi.ac.in

The no trade principle and the characterization of compact beliefs

Ngai-Ching Wong¹

Abstract: We establish the no trade principle, i.e., the no trade theorem and its converse, for any dual pair of bet and extended belief spaces, defined on a given measurable space. A key condition is that, except perhaps one of the agents, everyone else has (weak*) compact sets of beliefs. We find out that in most of the models of uncertainty adopted in the economic literature, roughly speaking, the epistemic statement that an agent has a compact set of beliefs is equivalent to the economic statement that he has an open cone of positive bets. This improves our understanding of what compactness actually means within an economic context.

¹ Department of Applied Mathematics, National Sun Yat-sen University
Kaohsiung, 80424, Taiwan
Email: wong@math.nsysu.edu.tw

On the multi-source Weber problem

Nguyen Dong Yen¹

Abstract: Several fundamental qualitative properties of the problem of minimizing the sum of the weighted minima of the Euclidean distances from the demand points to the facilities, which is called the multi-source Weber problem and also known as the clustering problem with Euclidean norms, are presented in this talk. The obtained results deepen our knowledge of the multi-source Weber problem and can be used to analyze the available methods for finding global solutions and local solutions to the problem.

¹ Department of Numerical Analysis and Scientific Computing,
Institute of Mathematics, Vietnam Academy of Science and Technology
18 Hoang Quoc Viet, 10307 Cau Giay, Hanoi, Vietnam
Email: ndyen@math.ac.vn

An extragradient algorithm for quasiconvex equilibrium problems without monotonicity

Le Dung Muu¹, Le Hai Yen²

Abstract: We attempt to provide an algorithm for approximating a solution of the quasi-convex equilibrium problem that was proved to exist by K. Fan 1972. The proposed algorithm is an iterative procedure, where the search direction at each iteration is a normal-subgradient, while the step-size is updated avoiding Lipschitz-type conditions. The algorithm is convergent to a ρ -quasi-solution with any positive ρ if the bifunction f is semistrictly quasiconvex in its second variable, while it converges to the solution when f is strongly quasiconvex. Neither monotonicity nor Lipschitz property is required. The main subprogram needed to solve at each iteration is a proximal regularized minimization problem whose objective function is the sum of a quasiconvex function and the one $\|\cdot\|^2$. We also discuss several cases where this global optimization problem can be solved efficiently.

¹ TIMAS, Thang Long University and Institute of Mathematics,
Hanoi 10307, Vietnam
Email: ldmuu@math.ac.vn

² Department of Optimization and Control,
Institute of Mathematics, Vietnam Academy of Science and Technology
18 Hoang Quoc Viet, 10307 Cau Giay, Hanoi, Vietnam
lhyen@math.ac.vn

LIST OF PARTICIPANTS

Duong Thi Viet An

Thai Nguyen University of Sciences,
Vietnam
andtv@tnus.edu.vn

Cung The Anh

Hanoi National University of
Education, Vietnam
anhctmath@hnue.edu.vn

Lam Quoc Anh

Can Tho University,
Vietnam
quocanh@ctu.edu.vn

Pham Ngoc Anh

Posts and Telecommunications
Institute of Technology, Vietnam
anhpn@ptit.edu.vn

Tran Thi Van Anh

Hanoi Pedagogical University 2,
Vietnam
tranthivananh@hpu2.edu.vn

Do Tuan Anh

Hanoi Pedagogical University 2,
Vietnam
dotuananh@hpu2.edu.vn

Nguyen Thi An

National Sun Yat-sen University,
Kaohsiung, Taiwan
nguyenthian@g-mail.nsysu.edu.tw

Imran Ali

Koneru Lakshmaiah Education
Foundation, India
imranali@kluniversity.in

Samir Adly

University of Limoges
870 60 Limoges, France
samir.adly@unilim.fr

Qamrul Hasan Ansari

Aligarh Muslim University
Aligarh 202 002, India
qhansari@gmail.com

Tran Van Bang

Hanoi Pedagogical University 2,
Vietnam
tranvanbang@hpu2.edu.vn

Feeroz Babu

Chandigarh University,
Mohali Punjab 140413, India
firoz77b@gmail.com

Nguyen Xuan Duy Bao

University of Science, Ho Chi Minh City,
Vietnam
nxdbao@gmail.com

Ramakant Bhardwaj

Amity University KolkataWB,
West Bengal 700135, India
rkbhardwaj100@gmail.com

Nguyen Thi Binh

Hanoi Pedagogical University 2,
Vietnam
nguyenthibinh@hpu2.edu.vn

Bui Kien Cuong

Hanoi Pedagogical University 2,
Vietnam
bukiencuong@hpu2.edu.vn

Lam Van Day

Nam Can Tho University,
Vietnam
lvdaz@nctu.edu.vn

Ha Tuan Dung

Hanoi Pedagogical University 2,
Vietnam
hatuandung@hpu2.edu.vn

Nguyen Trung Dung

Hanoi Pedagogical University 2,
Vietnam
nguyentrungdung@hpu2.edu.vn

Pham Thanh Duoc

Can Tho University of Technology,
Vietnam
ptduoc@ctuet.edu.vn

Nguyen Phuong Dong

Hanoi Pedagogical University 2,
Vietnam
nguyenphuongdong@hpu2.edu.vn

Nguyen Tuan Duong

National Sun Yat-sen University,
Taiwan
tdnguyen@nsvsu.edu.tw

Bui Van Dinh

Le Quy Don Technical University,
Vietnam
vandinb@lqdtu.edu.vn

Nguyen Bang Giang

Hanoi University of Civil Engineering,
Vietnam
Giangnb@huce.edu.vn

Duong Thi Ha

Hanoi Pedagogical University 2,
Vietnam
duongthiha@hpu2.edu.vn

Nguyen Hai Ha

University of Transport
and Communications, Vietnam
hanguyenhai@utc.edu.vn

Pham Thi Hong Hanh

Hanoi Pedagogical University 2,
Vietnam
hanhpth@hpu2.edu.vn

Nguyen Van Hao

Hanoi Pedagogical University 2,
Vietnam
nguyenvanhao@hpu2.edu.vn

Dinh Nho Hao

Institute of Mathematics, VAST,
Vietnam
hao@math.ac.vn

Pham Thanh Hieu

Thai Nguyen University of
Agriculture and Forestry, Vietnam
phamthanhhieu@tuaf.edu.vn

Phong Thi Thu Huyen

Institute of Mathematics, VAST,
Vietnam
ptthuyen@math.ac.vn

Pham Thi Huong

Hanoi Pedagogical University 2,
Vietnam
phamthihuong@hpu2.edu.vn

Nguyen Thi Thu Huong

Hanoi University of Science and
Technology, Vietnam
huong.nguyenthithu3@hust.edu.vn

Duong Thi Hong

Thai Nguyen University of Sciences,
Vietnam
hongdt@tnus.edu.vn

Dao Thi Hoa

Hanoi Pedagogical University 2,
Vietnam
daothihoa@hpu2.edu.vn

Nguyen Huy Hung

Hanoi Pedagogical University 2,
Vietnam
nguyenhuyhung@hpu2.edu.vn

Nguyen Quang Huy

Hanoi Pedagogical University 2,
Vietnam
nqhuy@hpu2.edu.vn

Le Van Hien

Hanoi National University of,
Education, Vietnam
hienlv@hnue.edu.vn

Monirul Islam

Aligarh Muslim University,
Uttar Pradesh 202001, India
monirul.amu@gmail.com

Bui Trong Kien

Institute of Mathematics, VAST,
Vietnam
btkien@math.ac.vn

Do Van Kien

Hanoi Pedagogical University 2,
Vietnam
dovankien@hpu2.edu.vn

Pham Duy Khanh

Ho Chi Minh City University of,
Education, Vietnam
khanhpd@hcmue.edu.vn

Huynh Khanh

Institute of Mathematics, VAST,
Vietnam
hkhanh@math.ac.vn

Phan Quoc Khanh

Ton Duc Thang University,
Vietnam
pqkhanh@hcmiu.edu.vn

Do Sang Kim

Pukyong National University,
Busan, 48513, Korea
dskim@pknu.ac.kr

Phan Van Loc

Hanoi Pedagogical University 2,
Vietnam
phanvanloc@hpu2.edu.vn

Duong Thi Luyen

Hanoi Pedagogical University 2,
Vietnam
duongthiluyen@hpu2.edu.vn

Dinh The Luc

University of Avignon,
France
dtluc@univ-avignon.fr

Vo Si Trong Long

University of Science, Ho Chi Minh City,
Vietnam
vstlong@hcmus.edu.vn

Nguyen Ngoc Luan

Hanoi National University of
Education, Vietnam
luannn@hnue.edu.vn

Vu Thi Thu Loan

Thai Nguyen University of
Agriculture and Forestry, Vietnam
vuloanb@gmail.com

Boris Mordukhovich

Wayne State University,
Detroit, MI 48202, USA
boris@math.wayne.edu

Le Dung Muu

Thang Long University,
Vietnam
ldmuu@math.ac.vn

Nguyen Huyen Muoi

Institute of Mathematics, VAST,
Vietnam
nhmuoi@math.ac.vn

Bui Ngoc Muoi

Hanoi Pedagogical University 2,
Vietnam
buingocmuoi@hpu2.edu.vn

Nguyen Hoai Nam

Hanoi University of Science and
Technology, Vietnam
nam.nguyenhoai@hust.edu.vn

Phan Thanh Nam

Quy Nhon University,
Vietnam
Phanthanhnam@qnu.edu.vn

Nguyen Mau Nam

Portland State University,
Portland, OR 97201, USA
mnn3@pdx.edu

Nguyen Thi Kieu Nga

Hanoi Pedagogical University 2,
Vietnam
nguyenthikieu@hpu2.edu.vn

Huynh Van Ngai

University of Quynhon,
Vietnam
ngai@yaho.com

Dang Thi Ngoan

Hanoi University of Natural Resources,
and Environment, Vietnam
dtngoan@hunre.edu.vn

Tran Van Nghi

Hanoi Pedagogical University 2,
Vietnam
tranvannghi@hpu2.edu.vn

Tran Ngoc Nguyen

Quy Nhon University,
Vietnam
tranngocnguyen@qnu.edu.vn

Vu Huu Nhu

Phenikaa University,
Vietnam
nhu.vuhuu@phenikaa-uni.edu.vn

Gyan Datt Pandey

K.M.P.G.College Kunda Pratapgarh,
India
gyandattpandey4@gmail.com

Vu Ngoc Phat

Institute of Mathematics, VAST,
Vietnam
vnphat@math.ac.vn

Hoang Xuan Phu

Institute of Mathematics, VAST,
Vietnam
hxphu@math.ac.vn

Nguyen Doan Phuoc

Hanoi National University of Science
and Technology, Vietnam
phuocnd-ac@mail.hut.edu.vn

Ta Duy Phuong

Institute of Mathematics, VAST,
Vietnam
tdphuong@math.ac.vn

Pham The Quan

Hanoi Pedagogical University 2,
Vietnam
phamthequan@hpu2.edu.vn

Dinh Hong Quang

Institute of Mathematics, VAST,
Vietnam
Quangdinhhong99@gmail.com

Arnd Rösch

Universität Duisburg-Essen,
Thea-Leymann-Straße 9 D-45127 Essen,
Germany
arnd.roesch@uni-due.de

Daya Ram Sahu

Banaras Hindu University
 Varanasi 221005, India
drsahudr@gmail.com

Pham Tien Son

Dalat University,
 Vietnam
sonpt@dlu.edu.vn

Doan Thai Son

Institute of Mathematics, VAST,
 Vietnam
ndyen@math.ac.vn

Nguyen Hai Son

Hanoi University of Science and
 Technology, Vietnam
son.nguyenhai1@hust.edu.vn

Vo Thanh Son

Ho Chi Minh City University of
 Industry and Trade, Vietnam
thanhson511@gmail.com

Pham Thanh Tam

Hanoi Pedagogical University 2,
 Vietnam
phamthanhtam@hpu2.edu.vn

Nguyen Nang Tam

Duy Tan University,
 Vietnam
nngtam@gmail.com

Vo Thanh Tai

An Giang University,
 Vietnam
vthanhtai@agu.edu.vn

Cao Thanh Tinh

University of Information Technology,
 Thu Duc city, Vietnam
tinhct@uit.edu.vn

Nguyen Minh Tung

Ho Chi Minh University of Banking,
 Vietnam
tungnm@hub.edu.vn

Nguyen Thi Toan

Hanoi University of Science and
 Technology, Vietnam
toan.nguyenthi@hust.edu.vn

Le Xuan Thanh

Institute of Mathematics, VAST,
 Vietnam
lxthanh@math.ac.vn

Le Chi Thanh

Hanoi University of Industry,
 Vietnam
mr.thanh.math@gmail.com

Le Quang Thuy

Hanoi University of Science and
 Technology, Vietnam
thuy.lequang@hust.edu.vn

Hoang Ngoc Tuan

Hanoi Pedagogical University 2,
 Vietnam
hoangngoctuan@hpu2.edu.vn

Nguyen Quoc Tuan

Hanoi Pedagogical University 2,
 Vietnam
nguyenquoctuan@hpu2.edu.vn

Pham Thanh Tuan

Hanoi Pedagogical University 2,
 Vietnam
phamthanhtuan@hpu2.edu.vn

Tran Van Tuan

Hanoi Pedagogical University 2,
 Vietnam
tranvantuan@hpu2.edu.vn

Tran Thanh Tung

Hanoi University of Science and
Technology, Vietnam
tung.tranthanh301202@gmail.com

Nguyen Van Tuyen

Hanoi Pedagogical University 2,
Vietnam
nguyenvantuyen83@hpu2.edu.vn

Izhar Uddin

Jamia Millia Islamia,
New Delh, India
izharuddin1@jmi.ac.in

Ngai-Ching Wong

National Sun Yat-sen University,
Kaohsiung, Taiwan
wong@math.nsysu.edu.tw

Nguyen Dong Yen

Institute of Mathematics, VAST,
Vietnam
ndyen@math.ac.vn

Le Hai Yen

Institute of Mathematics, VAST,
Vietnam
lhyen@math.ac.vn